

EFFECT OF RADIUS OF FILLET ON SHEAR STRESS FOR CYLINDRICAL  
SHAFT

NORSYAZWANI BT BADILAH

Report submitted in partial fulfillment of the requirements  
for the award of the degree of  
Bachelor of Mechanical Engineering with Automotive Engineering

Faculty of Mechanical Engineering  
UNIVERSITI MALAYSIA PAHANG

JULY 2012

## ABSTRACT

This project presents an investigation effect of the radius of the fillet on shear stress for cylindrical shaft by using finite element analysis. The shaft has the tendency to fail due to the inappropriate design of fillet radius. Hence, the best models need to design. 18 models were constructed with different ratio two diameters shaft and radius of fillet in SOLIDWORK. These models imported into PATRAN software to analyze the models with different given of the value of torque. In order to simulate torque, MPC RBAR was employed on model. AISI 4130 was selected material for all models. The value for Young Modulus of Elasticity = 206GPa and the Poisson Ratio = 0.3 was used in this analysis. The stress concentration factor was determined to compare the simulation result and equation. As expected, result showed that increased the radius of fillet of shaft reduced the value of shear stress. The ratio of two diameters shaft 1.09 was the best design compared to others because all cases produced the lowest value of shear stress. The value of shear stress was increase when the torque given was increased.

## ABSTRAK

Projek ini mempersembahkan mengenai kesan penyiasatan jejari kambi pada tegasan ricih untuk aci silinder dengan menggunakan analisis unsur terhingga. Aci mempunyai kecenderungan untuk gagal kerana reka bentuk yang tidak sesuai jejari kambi. Oleh yang demikian, model yang baik perlu di reka. 18 model telah dibina dengan nisbah yang berbeza dua diameter aci dan jejari kambi dalam SOLIDWORK. Kemudian, model-model yang diimport ke dalam perisian PATRAN penganalisis model dengan yang berlainan diberi nilai tork. Supaya dapat merangsangkan tork, MPC RBAR telah bekerja pada model. AISI 4130 telah dipilih bahan untuk model semua. Nilai Modulus Young = 206GPa Keanjalan dan Nisbah Poisson = 0.3 telah digunakan dalam analisis ini. Faktor tekanan kepekatan telah ditentukan untuk membandingkan hasil simulasi dan persamaan. Seperti yang dijangka, hasil menunjukkan bahawa meningkat jejari kambi aci mengurangkan nilai tegasan ricih. Nisbah dua diameter aci 1,09 adalah reka bentuk yang terbaik berbanding dengan yang lain kerana semua kes-kes yang menghasilkan nilai terendah tegasan ricih. Nilai tegasan ricih adalah meningkat apabila tork yang diberikan telah meningkat.

## TABLE OF CONTENTS

	<b>Page</b>
<b>SUPERVISOR’S DECLARATION</b>	iii
<b>STUDENT’S DECLARATION</b>	iv
<b>ACKNOWLEDGEMENTS</b>	vi
<b>ABSTRACT</b>	vii
<b>ABSTRAK</b>	viii
<b>TABLE OF CONTENTS</b>	ix
<b>LIST OF TABLES</b>	xi
<b>LIST OF FIGURES</b>	xii
<b>LIST OF SYMBOLS</b>	xiv
<b>LIST OF ABBREVIATIONS</b>	xv
 <b>CHAPTER 1      INTRODUCTION</b>	
 1.1          Background	1
1.2          Problem Statement	2
1.3          Objective	3
1.4          Scope of the Project	3
 <b>CHAPTER 2      LITERATURE REVIEW</b>	
 2.1          Introduction	4
2.2          Torsion of Shaft	4
2.2.1    Stress in elastic range	4
2.2.2    Stress Concentration Factor, $K$	6
2.3          Application of shaft	9
2.3.1    Drive Shaft	9
2.3.2    Elevator Drive Shaft	10
2.3.3    Turbine Engine Shaft	11
2.4          Finite Element Analysis	12
2.5          Material of shaft	14

### **CHAPTER 3      RESEARCH METHODOLOGY**

3.1	Introduction	16
3.2	Geometrical Modeling	18
3.4	FEA Analysis	20
	3.3.1 FEA Mesh Analysis	20
	3.3.2 Boundary Condition	21
	3.3.3 Material Properties	23

### **CHAPTER 4      RESULTS AND DISCUSSION**

4.1	Introduction	24
4.2	Stress Concentration, $K$	24
4.3	Result for $D / d$ equal to 1.09	26
4.4	Result for $D / d$ equal to 1.2	29
4.5	Result for $D / d$ equal to 1.33	31
4.6	Comparison result for different of the ratio of two diameters shaft	33

### **CHAPTER 5      CONCLUSION AND RECOMMENDATIONS**

5.1	Conclusion	36
5.3	Recommendations	37

<b>REFERENCES</b>	38
-------------------	----

### **APPENDICES**

A	Result for $D / d$ equal to 1.09	40
B	Result for $D / d$ equal to 1.2	46
C	Result for $D / d$ equal to 1.33	52

## LIST OF TABLES

Table No.		Page
2.1	Mechanical properties of steel (SM45C)	15
2.2	Properties of E-Glass/Epoxy, High strength Carbon Epoxy and HighModulus Carbon / Epoxy	15
3.1	The Parameter value of shaft design	19
3.2	Material properties of steel AISI4130	
4.1	The value of K is calculated by equation above in Tribology-ABC with the different of ratio of the two diameters and the constant of the ratio of fillet to smaller diameter	25
4.2	Result of the shear stress for $D/d$ is equal to 1.09	27

## LIST OF FIGURES

Figure No.		Page
2.1	Stepped shaft Geometry and Peterson original solutions for bending, tension and torsion	7
2.2	Effect of RC on stress distribution	8
2.3	Vehicle drive train	9
2.4	Elevator system inside building	10
2.5	Elevator drive system	11
2.6	A GE J85 turbine engine cut away so internal are visible. The shaft can be clearly seen.	12
2.7	Finite element mesh of the shaft	13
2.8	Finite Element Model and results of the stress analysis	14
3.1	Flow chart for methodology	17
3.2	Example of one complete design of cylindrical shaft	18
3.3	Dimension value for D, d and r	19
3.4	Step how to create automatic mesh	20
3.5	Example of complete mesh model	21
3.6	Step how to create MPC RBAR	21
3.7	Step to apply torque on shaft	22
3.8	Step how to fix on the shaft	22
3.9	Creating Nodal Moment and Creating Nodal Boundary Condition	23
4.1	Stress concentration factors for fillet in circular shafts	26
4.2	Result of shear stress for Torque = 150N.m	27
4.3	Shear stress versus torque for D/d is equal to 1.09	28
4.4	Shear stress versus radius of fillet	28
4.5	Result of shear stress for Torque = 200N.m	29
4.6	Shear stress versus torque for D/d is equal to 1.2	30

4.7	Shear stress versus radius of fillet	30
4.8	Result of shear stress for the lowest torque, $T = 50\text{N.m}$ Shear stress versus radius of fillet	31
4.9	Shear stress versus torque for $D/d$ is equal to 1.33	32
4.10	Shear stress versus radius of fillet	32
4.11	Shear stress versus radius of fillet for torque is equal $50\text{N.m}$	33
4.12	Shear stress versus radius of fillet for torque is equal $200\text{N.m}$	34
4.13	Shear stress versus torque at $r = 1.5\text{mm}$	34
4.14	Shear stress versus torque at $r = 9\text{mm}$	35



**LIST OF SYMBOLS**

$\tau$	Shear stress
$G$	Modulus of rigidity
$\gamma$	Shearing strain
$\rho$	Shearing stress in the shaft varies linearly with the distance
$c$	Circle of radius
$J$	Polar moment of inertia
$T$	Torque
$K$	Stress concentration factor
$\tau_{\max}$	Maximum shear stress
$r$	Radius of fillet
$D$	The largest diameter of shaft
$d$	The smallest diameters of shaft

## LIST OF ABBREVIATIONS

CAD	Computer-aided drafting
CAE	Computer-aided engineering
DOF	Degree-of-freedom
UMP	University Malaysia Pahang
FEA	Finite element analysis
FEM	Finite element method
SAE	Society of Automotive Engineers

**LIST OF APPENDICES**

<b>Appendix</b>	<b>Title</b>	<b>Page</b>
A	Result from PATRAN Analysis for Ratio of two diameters shaft, $D / d = 1.09$	41
B	Result from PATRAN Analysis for Ratio of two diameters shaft, $D / d = 1.2$	47
C	Result from PATRAN Analysis for Ratio of two diameters shaft, $D / d = 1.09$	53

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 BACKGROUND**

A shaft is the component of a mechanical device that transmits rotational motion and power. It is integral to any mechanical system in which power is transmitted from a prime mover, such as an electric motor or an engine, to other rotating parts of the system. There are many examples of mechanical systems incorporating rotating elements that transmit power: gear-type speed reducers, belt or chain drives, conveyors, pumps, fans, agitators, household appliances, lawn maintenance equipment, and parts of a car, power tools, machines around an office or workplace and many types of automation equipment. Torsional, bending and normal forces occur during the working of the shaft (Heisler, H. 1999). The forces cause a combination of normal and shear stresses which are applied cyclically on the shaft.

The stress concentrations can be determined in a shaft by various methods: mathematical methods, photoelasticity, Finite Element Method and others. There are researchers who determine stress by using the finite element method such as Bayrakceken, H. (2007) analyzed fatigue failure of driveshaft and universal joint yoke by the finite element method and Taylor, D. et al. (2011) also use the finite element method to study about two new methods for reducing the stress concentration features.

The geometry of a shoulder fillet has long been of interest to shaft designers and structural analysts. FEA was used to analyze additional fillet shaft geometries. Shouldered shafts with conical tapered sections tangent to a fillet radius were studied by Jallipalli. et al. (1997). Then the more general case of a shoulder with a tapered section that is not necessarily tangent to the fillet was study by Schmidt. et al. (2006).

In this project, the stress analysis of shaft was carried out by using finite element analysis. The different design of various steel shafts was created by using SOLIDWORK to determine the value of shear stress with variable torque. This study to prove that stress can be reduced through the use of a fillet.

## **1.2 PROBLEM STATEMENT**

A shaft is the component of a mechanical device that transmits rotational motion and power, usually of circular cross section either solid or hollow. There are different designs of shaft which can classify fillets into two categories namely sharp and well rounded, when a change in diameter occurs in a shaft to create a shoulder against which to locate a machine element, a stress concentration dependent on the ratio of the two diameters and on the radius in fillet is produced. Besides that, the shaft has the tendency to fail due to the inappropriate of fillet radius. Hence, there is need to construct model with the best design which is consider to the value of stress concentration,  $K$  so that the value shear stress will be decrease. Therefore, the main objective of this project is to examine stress analysis which to determine the shear stress for cylindrical shaft with various values of fillet by using FEA. The research work is focus on fillet in order to approve the stress can be reduced through the use fillet.

### **1.3 OBJECTIVE**

The objective of this project:

- a) To investigate effect of radius of fillet on shear stress for cylindrical shaft with various values of fillet by using finite element analysis.
- b) To compare shear stress the value of shearing stress with the different the ratio of the two diameters.

### **1.4 SCOPE OF THE PROJECT**

The scopes of this project as below are determined in order to achieve the objectives of the project:

- a) Construct the model of cylindrical shaft with different of the ratio of the fillet to the diameter shaft and the ratio two diameters of shaft by using SOLIDWORK.
- b) Determine the value of stress concentration,  $K$
- c) Analysis the shear stress with different model to prove that shear stress can be reduced through the fillet by using MSc PATRAN software.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

This chapter will explain more about the information collected related study on effect radius of fillet on shear stress for cylindrical shaft and provide additional information and relevant fact. It cover are of interested subtopic such as torsion of shaft, stress concentration factor,  $K$ , application of shaft, finite element analysis, and material of shaft.

#### **2.2 Torsion of shaft**

Torsion occurs when any shaft is subjected to a torque. The torque makes the shaft twist and one end rotates relatives to the other inducing shear stress on any cross section.

##### **2.2.1 Stress in the elastic range**

From the Hooke's Law for shearing stress and strain.

$$\tau = G\gamma \quad (2.1)$$

Where  $G$  is the modulus of rigidity or shear modulus of the material. The shearing strain,  $\gamma$  at a distance  $\rho$  from the axis of the shaft as :

$$\gamma = \frac{\rho}{c} \gamma_{max} \quad (2.2)$$

Multiplying Eq 2.2 by  $G$ ,

$$G\gamma = \frac{\rho}{c} G \gamma_{max} \quad (2.3)$$

$$\tau = \frac{\rho}{c} \tau_{max} \quad (2.4)$$

The equation obtained shows that, as long as the yield strength is not exceeded in any part of circular shaft, the shearing stress in the shaft varies linearly with the distance,  $\rho$  from the axis of the shaft.

The sum of the moments of the elementary force exerted on any cross section of the shaft must be equal to the magnitude  $T$  of the torque exerted on the shaft. (Ferdinand P. Beer. et al. 2009)

$$\int \rho (\tau dA) = T \quad (2.5)$$

Substituting for  $\tau$  from (2.4) in to (2.5) :

$$T = \int \rho \tau dA = \frac{\tau_{max}}{c} \int \rho^2 dA \quad (2.6)$$

Where  $J$  is polar moment inertia of circle of radius,  $c$  is

$$J = \frac{1}{2} \pi c^4 \quad (\text{for solid shaft}) \quad (2.7)$$

So, the elastic torsion formula as :

$$\tau_{max} = \frac{Tc}{J} \quad (2.8)$$



Where ;

T = torque, (N.m)

c = circle of radius, (m)

J = polar moment of inertia, ( m<sup>4</sup> )

$\tau$  = shear stress, (N / m<sup>2</sup> )

### 2.2.2 Stress concentration factor, $K$

In the case of shaft with a change in the diameter of its cross section, however, stress concentrations will occur near the discontinuity, with the highest stresses occurring at the fillet of shaft. These stresses may reduced through the use of fillet and the maximum value of the shearing stress at the fillet can be expressed as

$$\tau_{max} = K \frac{Tc}{J} \quad ( 2.9 )$$

The factor of  $K$  depends only upon the ratio of the two diameters and the ratio of the radius of the fillet to the diameter of the smaller shaft. The geometry of a shoulder fillet has long been of interested to shaft designers and structural analyst. For years engineers relied on solutions estimated by (Pilkey, W.D. et al. 2008) as presented in Figure 2.1 in the form of graphical design charts. Figure 2.1 was used to help determine the  $K$  value of a shoulder fillet on a cylindrical shaft in bending, torsion and tension.

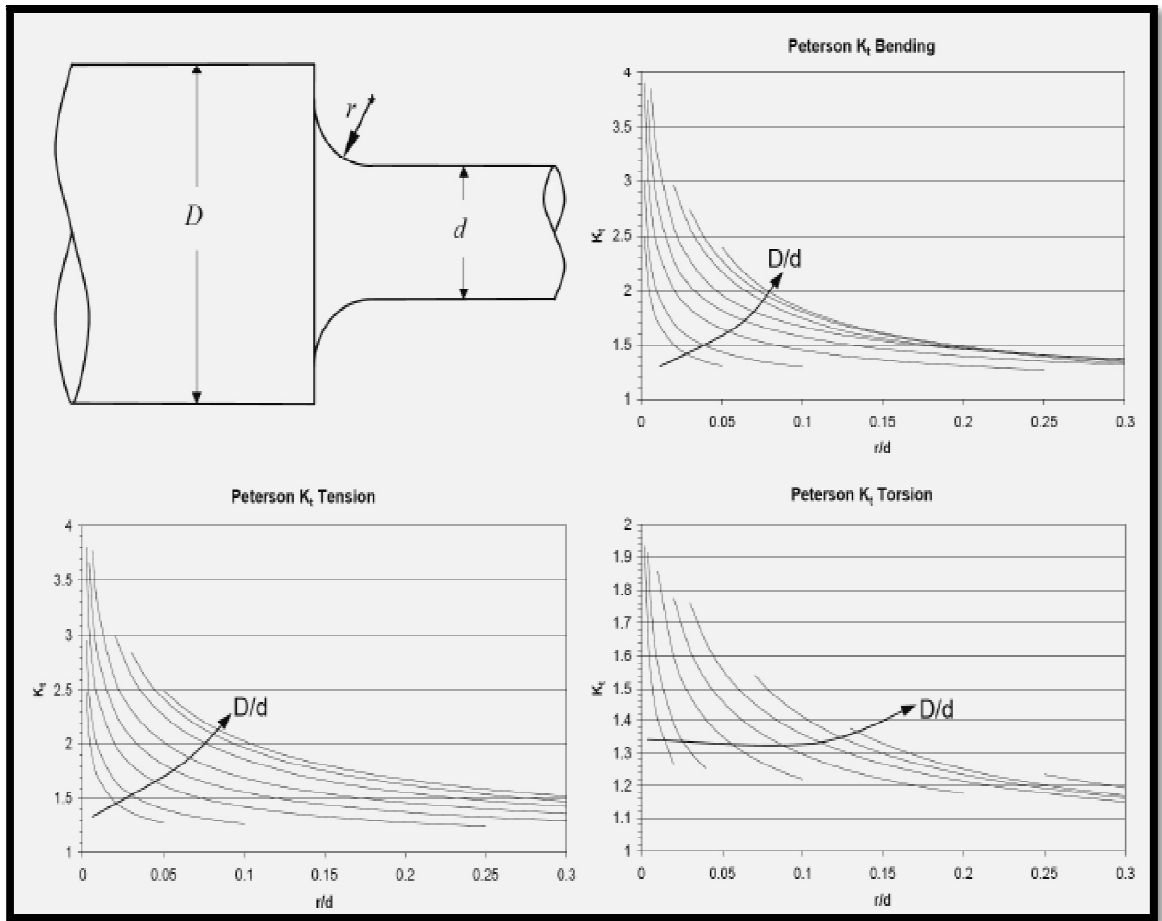


Figure 2.1 : Stepped shaft Geometry and Peterson original solutions for bending, tension and torsion

Sources : (Pilkey, W.D. et al. 2008)

Goksenli, A. et al. (2009) study about the failure analysis of an elevator drive shaft. This research found that the when the radius of curvature increase will reduce the shear stress. Figure 2.2 show the result of effect of radius of curvature on stress distribution.

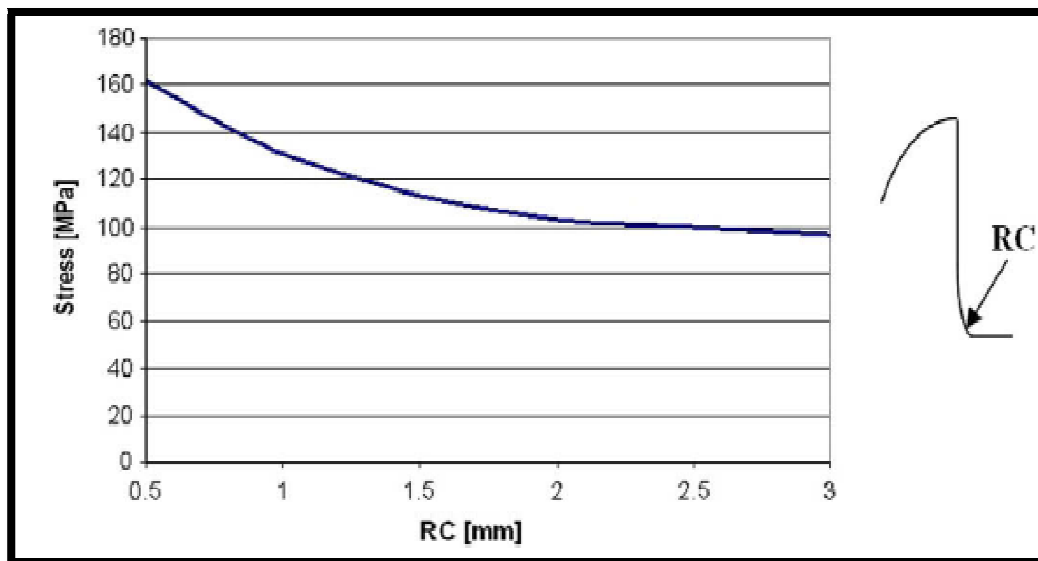


Figure 2.2 : Effect of RC on stress distribution

Source : Goksenli, A. et al. (2009)

## 2.3 APPLICATION OF SHAFT

There are many examples of mechanical systems incorporating rotating elements that transmit power: gear-type speed reducers, belt or chain drives, conveyors, pumps, fans, agitators, household appliances, lawn maintenance equipment, and parts of a car, power tools, machines around an office or workplace and many types of automation equipment

### 2.3.1 Drive Shaft

The torque that is produced from the engine and transmission must be transferred to the rear wheels to push the vehicle forward and reverse. The drive shaft must provide a smooth, uninterrupted flow of power to the axles. The driveshaft and differential are used to transfer this torque. Figure 2.3 showed the vehicle drive train which is showed the driveshaft is the connection between the transmission and the rear axle of the car.

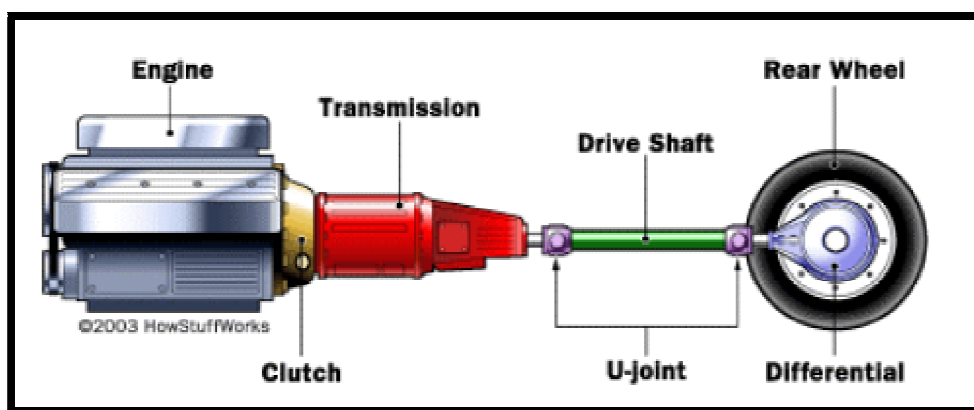


Figure 2.3 : Vehicle drive train

Source : Chad, K. et al. (2004)

### 2.3.2 Elevator Drive shaft

Goksenli, A. et al. (2009) study failure analysis of an elevator drive shaft. Elevator drive system is mounted at the bottom of building in Figure 2.4. Torque which is produced by an electric motor is transmitted by a worm gear to shaft. The shaft rotates the pulley by a key.

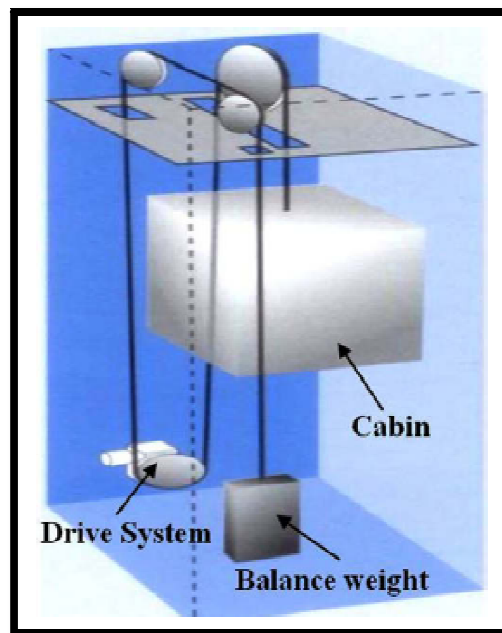


Figure 2.4 : Elevator system inside building

Source : Goksenli, A. et al. (2009)

Figure 2.5 showed the elevator drive system which is the shaft is supported in three points in form of journal bearings.

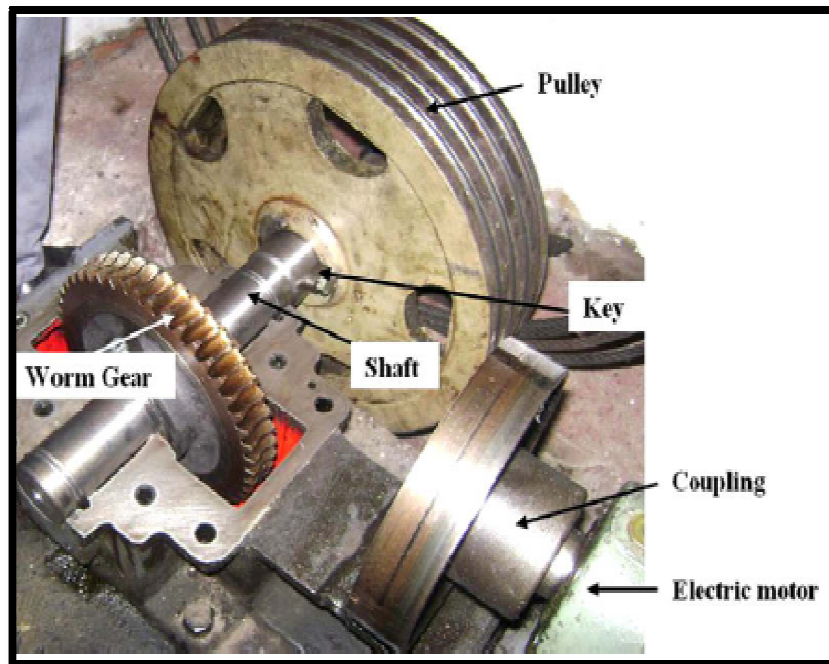


Figure 2.5 : Elevator drive system

Source : Goksenli, A. et al. (2009)

### 2.3.3 Turbine Engine Shaft

A turbine engine shaft transfer torque created by the engine's the turbine to the engine's compressor. In this process, the turbine shaft incurs rotational shear loads, bending loads, and a thermal gradient. Figure 2.6 shows the turbine engine.

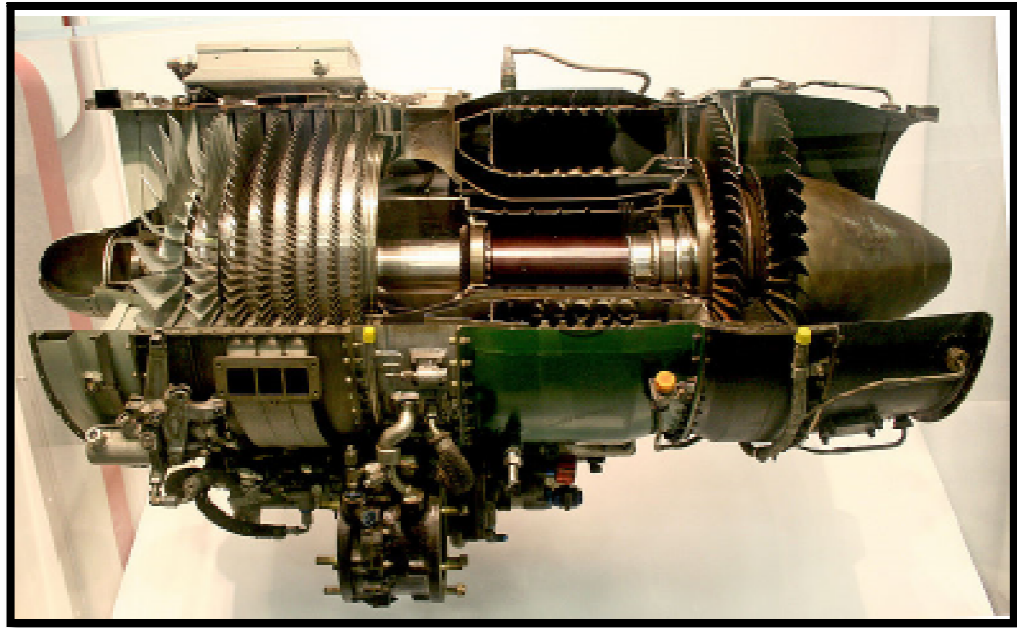


Figure 2.6 : A GE J85 turbine engine cut away so internal are visible. The shaft can be clearly seen.

Source : Steven Dallas Townes Jr. (2009)

## 2.4 FINITE ELEMENT ANALYSIS

Most recently, advance in technology have produced computers that are capable of quickly and efficiently running simulated analyses of geometrically complex components. The finite element method (FEM) has been established for many shaft as a means of analytically determining stresses but its usage has been somewhat limited by the processing power available. Goksenli, A. et al. (2009) showed that it was possible to accurately analyses an elevator drive shaft with the FE method. Their investigation was carried out with 3D model. A precise geometrical model of the shaft was built up and then imported to ANSYS to analyse the stress occurring at the keyway corner. Figure 2.7 shows the finite element mesh of the shaft.

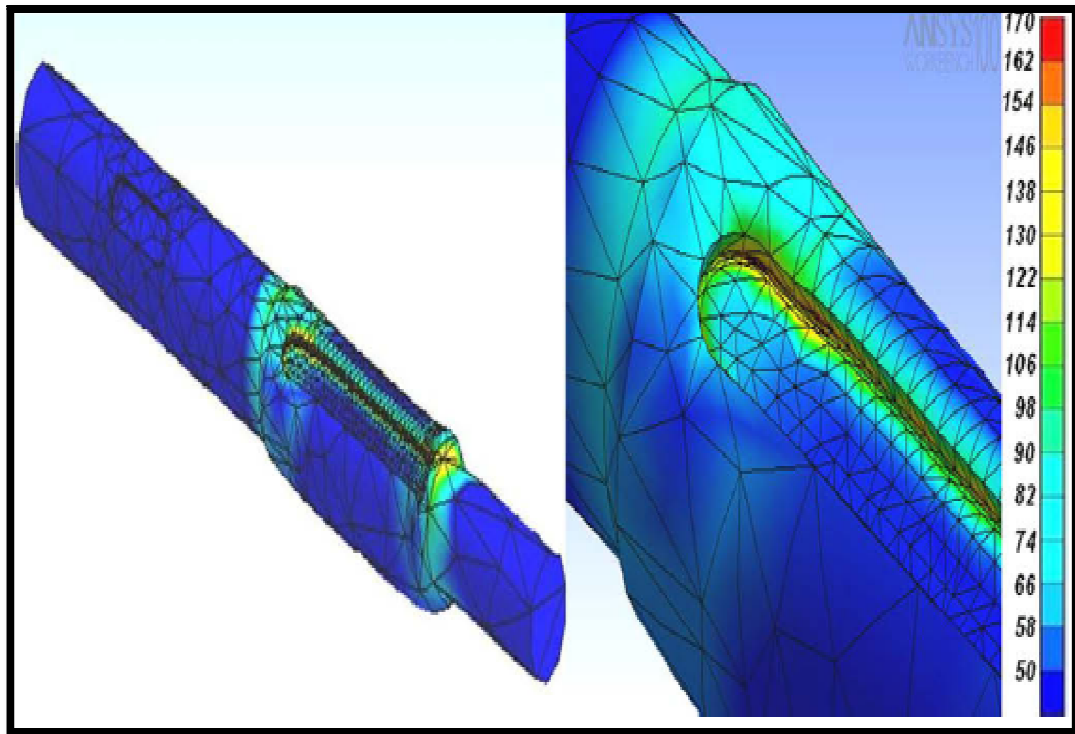


Figure 2.7 : Finite element mesh of the shaft

Source : Goksenli, A. et al. (2009)

Bayrakceken, H. (2007) research about two cases of failure in the power transmission system in vehicle. Finite element analysis technique is used to determine the stress distribution at the failed section and to obtain the best design. After the construction of the geometric model, a static stress analysis is carried out by entering the obtained mechanical properties of the material ( Elastic Modulus = 205GPa, Poisson Ratio = 0.29). The load are lateral bending load of weight of the full car (2500N) and torsional moment for turning of the wheel 100N.m. The mesh consist of 11,486elements and 20,612 nodes. Boundary condition are applied at the bearing and geared coupling locations. Figure 2.8 show that the highest stress occur at the fillet region of the failed cross section.